1. Rationale for the Study:
   - Why university mathematics?
     • Importance of STEM education
     • School Math
     • Existing Research
     • Transition from School to University Math
   - Why an international study?
     • Internalization and globalization
     • Diversity of university mathematics faculty

2. Definitions

3. Research Questions

4. Why “beyond the word”?
   • Multimodality of teaching practices in the university mathematics classroom
   • Chalk Talk

5. Theoretical and Analytical Approaches

6. Methods

7. Selected Findings

8. Discussion

9. Conclusions and Implications
Why University Mathematics?

• Mathematics is often referred to as a gate keeper – limiting career options by creating obstacles (at times insurmountable) for students pursuing university degrees in, for example, engineering, economics, or science.

• There has been widespread international recognition, supported by research, that math skills amongst high school graduates have been deteriorating (e.g. Kershaw, 2010; LeFevre, 2010; Zwaagstra, 2011).

• Universities are reporting higher failure rates in entry-level mathematics courses leading to increased costs for both the institutions, which are attempting to provide support (Stevenson & Zweier, 2011; Zwaagstra, 2011), and students, who are either dropping out of or persisting in programs for longer periods of time (e.g., Fox, 2005).
Why University Mathematics? (continued)

- The role of science and technology in increasingly globalized contexts is expanding:
  - Science, technology, engineering, and mathematics (STEM) higher education => societies’ development of advanced knowledge in these fields => facilitates positive social and economic externalities through innovation
  - **the importance of STEM education at all levels**
    - In the U.S., elementary and secondary students still lag behind international averages in mathematics
      » Without a firm foundation in Grades K–12, students cannot expect to excel in postsecondary STEM courses. (Indicators, 2012)
Why University Mathematics? (continued)

• In Canada
  – Math scores in elementary and secondary schools have been trending down across the country. (The Globe and Mail, Sept 25, 2013)

• Increasing numbers of students across the world are earning advanced degrees in the natural sciences and engineering
  – Especially among minorities and women

• Postsecondary educational attainment in the natural sciences and engineering is shifting eastward:
  – In 2008, 56% of the world’s undergraduate engineering degrees were awarded in Asia. (Indicators, 2012)

• STEM International competition and partnerships continue to shape global capabilities

• Focus is shifting from North America
Existing Research

• Limited research of pedagogical practices in the university mathematics classroom (cf. Speer, Smith, & Horvath, 2010)

• A view of university mathematics pedagogy and its pervasive use of lectures as based on a transmission model (Barnes, 1992), which undervalues pedagogic interaction (e.g., Bligh, 1998; Godon, 2008; Greenberg & Williams, 2008; Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011)

• Mathematics teaching at university needs to be changed (e.g., Gibbs & Habeshaw, 1992).

• A small but growing body of research which challenges this view (e.g., Artemeva & Fox, 2011; Jamison, 2000; Pritchard, 2010; Rodd, 2003; Zwaagstra, 2010)

• Evidence that university mathematics teaching serves as an initiation into practices central to the discipline
  – Changes to central pedagogical practices in university mathematics without appropriate research may well prove harmful to students (Pritchard, 2010, p. 610).

Such contrasting views suggest a critical gap in our understanding of university mathematics pedagogic practices and interactions between teaching and learning mathematics in university.
• A Canadian study (Dame, 2012):
  – A significant proportion of students beginning entry level undergraduate mathematics courses do not demonstrate the high levels of preparation required to succeed
    • Optional additional remediation is not addressing students' needs
    • Struggling students do not frequently engage with departmental supports offered
“. . . many people see mathematics only as a collection of arcane rules for manipulating bizarre symbols something far removed from speech and writing”

- Most elementary and high school are procedural courses focusing on techniques for working with numbers, symbols, and equations (symbolic manipulation).
  - Formal technique is important, but
    - Formulae are not ends in themselves but derive their real importance only as vehicles for expression of “deeper mathematical thoughts”
    - “Conceptual mathematics courses focus on proof and argument”
    - More advanced courses are concerned not just with manipulating symbols and solving equations but with “understanding the interrelationships among a whole host of sophisticated concepts” (Jamison, 2000, p. 45)

- There “is a difficult but crucial leap for students to make in transitioning from rudimentary to advanced mathematical thinking” (p. 46)
Why an International Study?

• Early stages of our research:
  – Mathematics departments not only in Canada and the United States but also in many other countries, for example, Australia, Israel, Spain, Sweden, the United Kingdom, all of which are considered in this study, reflect common internationalization and globalization trends and are currently characterized by extraordinary cultural and linguistic diversity (Artemeva & Fox, 2010).
– List of the Fields Medalists:

• Many award recipients speak first languages (L1) or were educated in languages other than the language of the country in which they reside and teach

  – Out of 13 award recipients in the period 1990 to 2010 who are currently working in U.S. universities, 11 were born and educated outside of the United States, with 8 of these speaking L1s other than English, i.e., Cantonese, French, Hebrew, Russian, and Vietnamese (International Mathematical Union, 2011).
Why an International Study? (continued)

• The need to investigate the global context is based on the growing internationalization and globalization trends (de Wit, 2002; Knight, 2004) in science, engineering, and university education
  – North America has long benefited from the participation of large numbers of foreign-born scientists, engineers, and mathematicians in the workforce
  – Growing “international labour mobility” (Knight, 2004, p. 6) among university faculty appears to be a feature of the university mathematics departments across the world
Definitions (Artemeva & Fox, 2011)

- Teaching within a discipline:
  - “a kind of rhetorical accomplishment” (Fox, 2009, p. 29), which is realized in the social practices and discursive accounts of key stakeholders

- Acknowledging that the terms globalization and internationalization are contentious and often confusing, Knight (2004) observes that
  - internationalization “emphasizes the notion of nation”
  - globalization “refers to worldwide in scope and substance” (p. 8).
Definitions (continued) (Artemeva & Fox, 2011)

• Global context
  – the teaching of mathematics in university classrooms around the world (focusing mostly on lectures, but also on problem solving sessions)

• Local context
  – the teaching of mathematics in a unique classroom, which is situated within cultural, institutional, national, linguistic, social, or nested, contexts (cf. Maguire, 1994)

• Culture
Research Questions

• What are the pedagogical practices of university mathematics teachers in the current environment of globalization and internationalization of higher education?

• Are these practices realized differently by veteran and novice teachers in different contexts?
Why “beyond the word”?  

Across all the observed contexts, mathematics teachers were—often simultaneously—engaged in the same pedagogical practices through the use of multiple resources (the “chalk talk”) (Artemeva & Fox, 2010, 2011; Fox & Artemeva, 2011):

- Writing on a board (chalk, white, electronic, etc.) (central practice)
  - mathematical symbols,
  - graphs,
  - diagrams,
  - prose, etc.
- Articulating what is being written
- Talking out loud about what has been written (meta-discourse)
- Using a variety of gestures to
  - indicate relationships, signal references, highlight key issues, etc. written on the board
  - elicit students’ responses and engage them in discussion (Artemeva & Fox, 2011; Fox & Artemeva, 201; Fograty-Bourget, 2013)
- Talking to and/or with students,
- Moving in space (using typified moves)
  - Turning to students and asking questions
- Referring to problem sets, textbook chapters, websites, tests, etc. (not necessarily physically present in the classroom)
- Referring to their lecture notes (present in class), etc.
Pedagogical Practices of University Mathematics Teachers: “Chalk talk”
A university mathematics professor (Borovik, 2008) wrote in his on-line blog that a university mathematics teacher is not just conveying information, he or she teaches [students] to think mathematically, and teaches by example, *in real time*. It is crucially important to be *in full control of timing and tempo* of the narrative. If a lecture involves calculations (and they are inevitable in most mathematical disciplines), it is crucially important to let students *feel the subtle play of rhythms*, emphasize switches and branch points in the procedure, highlight recursion and reduction to simpler cases. (24 September, para 4) (italics added)
Theoretical & Analytical Frameworks

- Rhetorical Genre Studies (RGS)
- Situated Learning and Communities of Practice (CoP)
- Multimodality (mediated discourse theory)
- Gesture Theory
Rhetorical Genre Studies (RGS)

• Genre
  – Relatively stable type of utterances (Bakhtin, 1986, p.60) used by individuals to participate in social interactions
    • Utterance as “a link in the chain of speech communication” (p. 91)
  – ‘Social action’ (Miller, 1984)
    • Typified rhetorical action
      – Develops in response to a recurrent social situation
      – Forms the social situation (Bawarshi, 2000; Paré & Smart, 1994)
  – Stabilized for now, stabilized enough (Schryer, 1993)
  – Constraining yet enabling

• RGS is
  – Compatible and complementary with theories of situated learning and communities of practice (CoPs) (Artemeva, 2008; Lave & Wenger, 1991; Wenger, 1998)
  – Useful for understanding how genres are learned by novices in different contexts
Definition of Genre

‘Constellations’ (Campbell & Jamieson, 1979) of overtly recognized rhetorical conventions and tacit and improvisational strategies, triggered by the interaction between an individual’s socialization and an organization (Schryer, 2000; Schryer & Spoel, 2005)

– where in this study organization is understood as a discipline in a university, embedded within a larger cultural-historical educational context (Artemeva & Fox, 2010)
Situated Learning and CoPs

- Theory of situated learning (e.g. Lave, 1991; Lave & Wenger, 1991; Wenger, 1998) views
  - learning as a social process involving an individual as an active participant in the practices of social communities (Wenger, 1998)
- Community of Practice is the central unit of analysis in the situated learning perspective
  - CoP is formed by people engaged in a process of collective learning in a domain of shared goals, values, and beliefs
    - not necessarily a physical domain, but rather a shared domain of interest and human endeavor (Wenger, 1998; 2006)
A growing recognition that what counts as writing is much more complex than was once believed;

Focusing on written text in isolation may reduce and even distort its very nature (e.g., Lemke, 1998; O’Halloran, 1998).

- consistent with the well-established notions of writing as social action (Cooper & Holzman, 1989) and of genres as typified rhetorical actions (Miller, 1984)

“The analysis of the mathematical pedagogical discourse must necessarily take into account . . . the shifts between the written/spoken modes and the shifts between the language, symbolism and visual display” (O’Halloran, 2005, p. 206).
Multimodality

- Multimodality theory and theory of mediated action take into account non-verbal elements of communication alongside written and spoken language (Bearne, 2009)
- Multimodal text is a combination of image, sound, movement, writing, face to face meetings, or performative space
  - This view allows for a holistic analysis without presupposition that any one mode is inherently more important than another (Norris, 2004)
Multimodality

... multimodality has always and everywhere been present as representations are propagated across multiple media and as any situated event is indexically fed by all the modes present... In this sense, all genres are irremediably multimodal; the question then becomes what particular configurations of multimodality are at work.

(Prior, 2009, p. 27)
In mediated discourse theory (Norris & Jones, 2005) “a language is a mode which social actors use to act in the world, but it is a mode that may essentially differ in property from other modes” (Norris, 2012, p. 223)

- “The underlying structure of mediated discourse theory is a theory of action” (p. 223)
  - This is consistent with the Rhetorical Genre Studies (RGS) view of “genre as social action” (Miller, 1984; Russell, 2012)
As researchers working within the mediated discourse theory tradition,

- we consider modes “of sound, visual layering, movement” (Jewitt as cited in Norris, 2012, p. 5), writing, and so on from a *cinematic* perspective (Fox & Artemeva, 2011) by viewing the embodied genre of chalk talk as a *layered multimodal performance*

- Among layered modes used to enact the genre of chalk talk, the primary modes are *writing (on the board), speaking, and movement*
  - The notion of layering is consistent with emergent and developing perspectives on disciplinary and professional written discourse as an embodied and multimodal phenomenon
In Gesture theory, “the term ‘utterance’” refers to any “ensemble of action that counts for others as an attempt by an actor to ‘give’ information of some sort” (Kendon, 2010, p. 7)

– Communication can occur through speech or through “visible bodily action or . . . [through] combinations of these two modalities” (p. 7)
‘Chalk Talk’ Genre of Math Teaching
(Artemeva & Fox, 2010, 2011; Fox and Artemeva, 2011)

• The central pedagogical genre of the undergraduate mathematics classroom

• The *way* university mathematics lectures happen in different national, cultural, linguistic, and institutional contexts

• A genre comprised of a variety of complex interweaving and overlapping *verbal* and *non-verbal* features recurring across global and local contexts
Method: A multi-method, multi-site, multi-case study

Sites:
11 Canadian and international sites, including national, linguistic, educational, institutional, disciplinary, and classroom contexts (these are our nested contexts [Maguire, 1994]).

Participants: 50 participants:
12 females and 38 males;
16 first languages; 8 languages of instruction
<table>
<thead>
<tr>
<th>Status</th>
<th>#</th>
<th>Gender</th>
<th>First Language Background</th>
<th># of speakers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Female</td>
<td>Armenien</td>
<td>1</td>
</tr>
<tr>
<td>Professor</td>
<td>36</td>
<td>7</td>
<td>Chinese</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>English</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>French</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>German</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hindi</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hebrew</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Polish</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Romanian</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Russian</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Spanish</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Swedish</td>
<td>1</td>
</tr>
<tr>
<td>Post-Doctoral Fellow</td>
<td>8</td>
<td>5</td>
<td>English</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hebrew</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hungarian</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Italian</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Persian</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Russian</td>
<td>1</td>
</tr>
<tr>
<td>Teaching Assistant</td>
<td>6</td>
<td>6</td>
<td>Arabic</td>
<td>1</td>
</tr>
<tr>
<td>(Graduate Student)</td>
<td></td>
<td></td>
<td>English</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Chinese</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>French</td>
<td>1</td>
</tr>
</tbody>
</table>
In order to analyze the complex pedagogical genre of chalk talk, we needed a “multimodal record” (Jewitt, 2012, p. 6). Video provides such a multimodal record in which talk is kept in context and all modes are recorded … [enabling] researchers to rigorously and systematically examine resources and practices through which participants . . . build their social activities and how their talk, facial expression, gaze, gesture, and body elaborate one another. (p. 6)
Data

- Video-recorded and transcribed lectures
- Audio recorded and transcribed one-on-one semi-structured interviews
  - Follow-up interviews
  - Follow-up e-mail communication
- Field notes (class observations and interviews)
- Written artefacts: published textbooks, lecture notes, website notes, etc.
Data Analysis

• Modified constructivist grounded theory (MGT) (e.g., Charmaz, 2000, 2002, 2006; Schryer & Spoel, 2005)
  – Computer-assisted thematic analysis of lecture and interview transcripts (NVivo)
  – Multimodal analysis of video recordings (Fox & Artemeva, 2011; Norris, 2008)
    • MGT analysis allows us to identify what *repeats* and what *differs* (Paré & Smart, 1994) in the enactment of the genre of chalk talk across sites and participants
    – Gesture analysis (Nvivo) (Fogarty-Bourget, 2013)
• Descriptive statistics
• Member checks (e.g., playing video-recordings to study participants for comment)
Audio Transcript Coding (Nvivo) (Fogarty-Bourget, 2013)
Multimodal Coding  (Multimedia feature in Nvivo)
(Fogarty-Bourget, 2013)
Among layered modes used to enact the genre of teaching undergraduate mathematics (chalk talk), the primary modes are writing (on the board), speaking, movement
Board Choreography

• While chalk talk is unfolding, teachers are continuously cognizant of the positioning of new writing on the board in relation to the chalk talk already inscribed.
  – We call this chalk board management choreography
• Board choreography is part and parcel of the genre of chalk talk.
  – As a veteran university mathematics teacher stressed,
    • A lot of thinking goes into, in the blackboard. You need to think what to keep, what to . . . erase, um, where to write it so that it would not get in your way . . . if you want to keep it for a while
• All of our participants commented that a mathematics classroom must have a large chalkboard:
  
  . . . ideally a good [class]room will have a huge wall of blackboard where you can move and where things can stay . . . for a long time. So it’s, the ability to go back to information that [was] . . . given 10, 20, almost 30 minutes before. . . . [because] in math . . . the density of information is incredible.

• Multiple sliding chalkboards require more complex choreography and thinking:
  
  [There are] . . . three boards that I can write on, . . . [that] I [will] use. So there’s a front board, a board behind it, and then the fixed board. And I think it’s best to write on the middle board first and slide that up, and then the second . . . the top board, and slide that up, and then the bottom board . . .
Mode of Movement

- Body positioning and shoulder line
- Focus
  - Directionality of gaze
- Gesture
- Facial Expression
Levels of Multimodal Analysis: Mode of Movement (Fox & Artemeva, 2011, p. 98)

**MODE:**
- Talking
- Writing
- Movement

**ELEMENT:**
- Positioning
- Focus

**COMPONENTS:**
- Teacher’s body line and shoulder orientation
- Teacher is looking at
  1. Notes
  2. Board
  3. Class (students)

1. Back to class (facing board)
2. 1/2 turn away from board --shoulders perpendicular to board and class. (facing board)
3. 3/4 turn away from board (facing class)
4. Back to board (facing class/students)
<table>
<thead>
<tr>
<th>Time*</th>
<th>Talking</th>
<th>Writing</th>
<th>Mode of Movement**</th>
<th>Focus</th>
<th>Illustration</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:03</td>
<td>YES</td>
<td>NO</td>
<td>Full front orientation to class</td>
<td>Class</td>
<td><img src="image1.png" alt="Illustration" /></td>
</tr>
<tr>
<td>00:09</td>
<td>YES</td>
<td>NO</td>
<td>180° turn; facing board</td>
<td>Board</td>
<td><img src="image2.png" alt="Illustration" /></td>
</tr>
<tr>
<td>00:11</td>
<td>NO</td>
<td>NO</td>
<td>Facing board</td>
<td>Glancing at notes momentarily</td>
<td><img src="image3.png" alt="Illustration" /></td>
</tr>
<tr>
<td>00:12</td>
<td>YES</td>
<td>YES</td>
<td>Facing board</td>
<td>Board; intermittently glancing at notes</td>
<td><img src="image4.png" alt="Illustration" /></td>
</tr>
<tr>
<td>00:27</td>
<td>YES</td>
<td>NO</td>
<td>Stepping back into a 90° half turn; shoulders perpendicular to the board</td>
<td>Board</td>
<td><img src="image5.png" alt="Illustration" /></td>
</tr>
</tbody>
</table>
Multimodal Analysis of Chalk talk

• The table
  – allows for the analysis and quantification of similarities and differences by providing an individual profile:
    • a record of the number and duration of intervals
    • allocation of time for the enactment of genre elements (body positioning, focus, etc.)
  – demonstrates the time-space (chronotopic [Bakhtin, 1986]) characterization of the typified and recurrent chalk talk components:
    • teacher’s body and shoulder orientation within the element of positioning and
    • directionality of teacher’s gaze within the element of focus
• A multimodal investigation of the strategies used by university teachers of mathematics to elicit responses from students
  
  – When “gestures and speech are employed together as partners in a single rhetorical enterprise” the speaker works to create an ‘ensemble’ of meaning (Kendon, 2004, p. 127)
Pointing, Index Finger Extended (IFE)

Example 1 (L1 English; Teaches in English, Veteran Teacher, North America)

- Pointing gestures are used to indicate an object, location, or direction
- IFE is used when a speaker singles out an object which is to be attended to as a particular individual object
Open Hand Supine (OHS) Palm Presentation (PP)

Example 2 (L1 English; Teaches in English, TA, North America)

- The Open Hand Supine (OHS) family of gestures has in common the feature of offering or giving, or showing readiness to receive something, whether a concrete item, or the content of what is being said.
- PP is typically used in coordination with a speaker introducing something he or she is about to say, or during an explanation, comment, or clarification of something the speaker has just said.
Outcomes of Multimodal Genre Analysis of Chalk Talk

Multimodal coding for the elements of positioning and focus within the movement mode, and gestures allowed us to

– observe the components as they unfold in the time-space (the chronotope [Bakhtin, 1986]) of the university mathematics lecture classroom

– identify and document similarities and differences in the multimodal enactment of the pedagogical genre of chalk talk
  • across contexts, and
  • among veteran and novice university teachers of mathematics
Conclusions: Chalk Talk as a Complex, Interactive Genre

• Chalk Talk
  – Disciplinary and professional genre of the international CoP of mathematicians
  – Meaningful disciplinary practice of teaching university mathematics
    • Helping students to enter the CoP as novices
  – Multiplicity of multimodal (verbal and non-verbal) patterns occurring and co-occurring simultaneously
    • cannot be teased apart, i.e., cannot be analyzed in isolation

• Contrary to some description of university mathematics teaching as “unengaged” and “unengaging,” university teachers of mathematics, observed in this study, were always engaged and always interacting with students
  – Disciplinary engagement and interaction
Implications and Directions for Future Research

Implications for

- Research:
  - Learning more about genres of teaching may not only
    - result in a better understanding of the relationships between global and local disciplinary communities of practice but also
    - increase our understanding of academic literacy and effective teaching and learning
  - Training novice university mathematics teachers in disciplinary pedagogical genres:
    - Mentorship within communities of practice
  - Critical view of disciplinary tradition and innovation

- Directions for Future Research:
  - Longitudinal studies of novices entering the international CoP of university mathematics teachers
  - Studies
    - of the interactions between teaching and learning mathematics in university
      » investigate and identify pedagogical practices and processes in university mathematics classrooms that engage students, promote their learning, and increase the impact of teaching on student learning in university mathematics
    - of the impact of technology on the university mathematics classroom
Thank you.

Any Questions?

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